Scaling the Hardening Behavior of Nonlinear Timoshenko Beams for the Design of Lattice Materials

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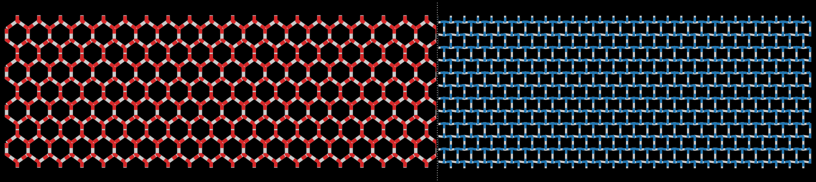

Gärtner, van den Boom, Weerheijm, and Sluys *Mechanics of Materials* 191 (2024)

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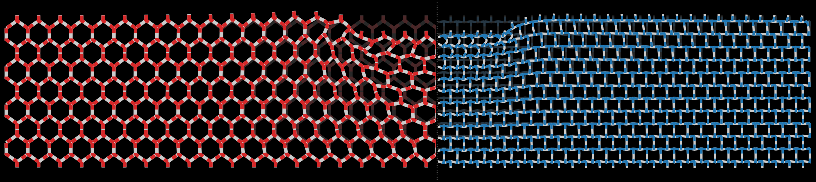

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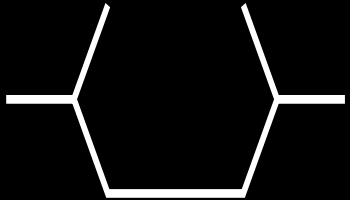
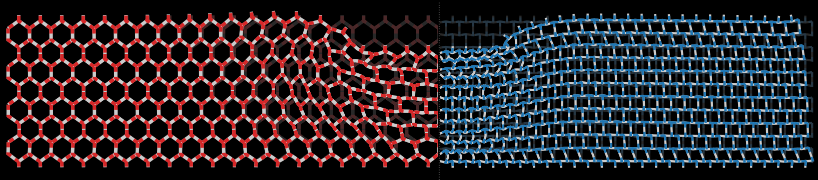
- Designing architectures requires fitting material models for all scales





Gärtner, van den Boom, Weerheijm, and Sluys *Mechanics of Materials* 191 (2024)

Scaling of plastic beam properties not fully explored

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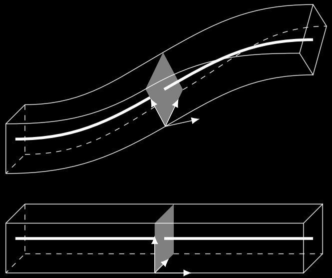

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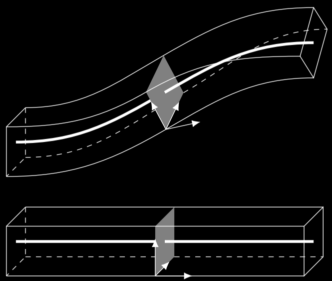

Smriti, Kumar, Großmann, and Steinmann *Mathematics and Mechanics of Solids* 24.3 (2018)
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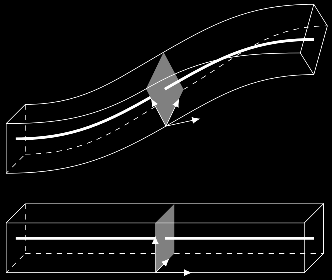

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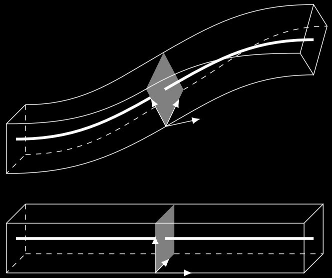
- The scaling of the hardening tensor is not fully explored



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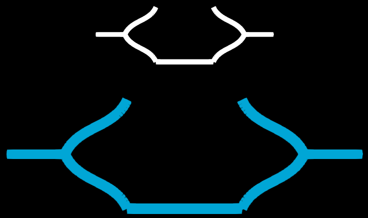

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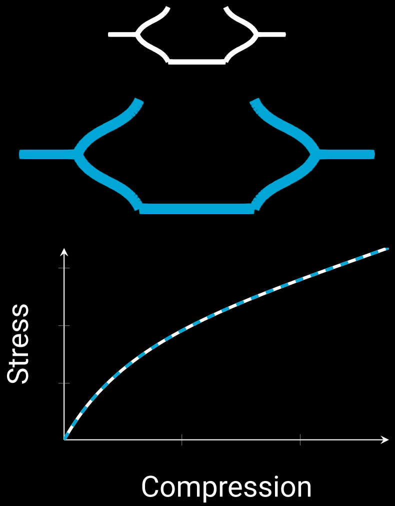

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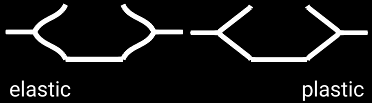

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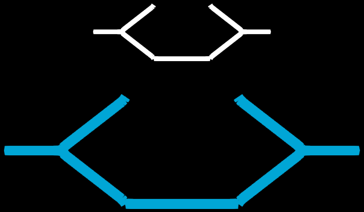

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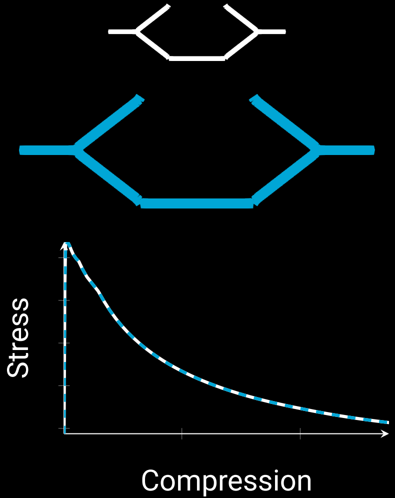

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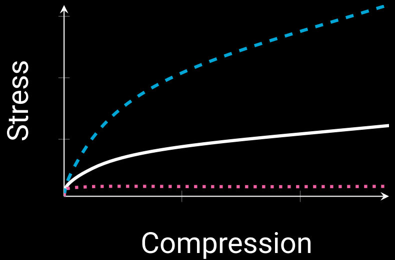

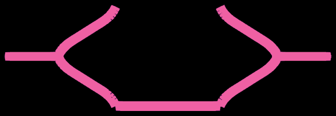

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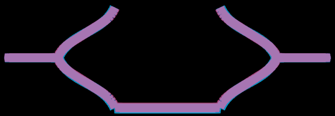

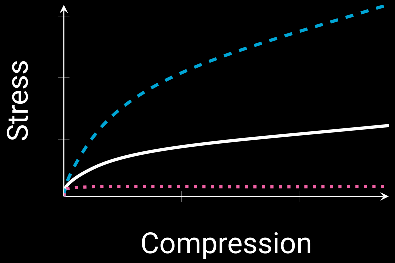

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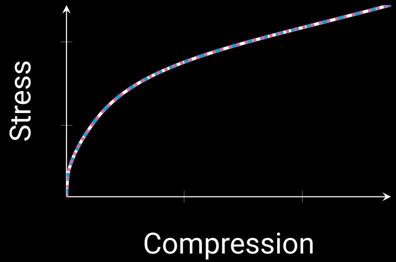

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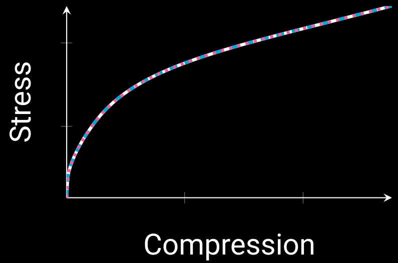

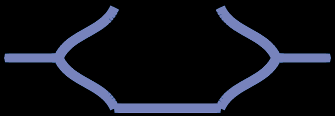

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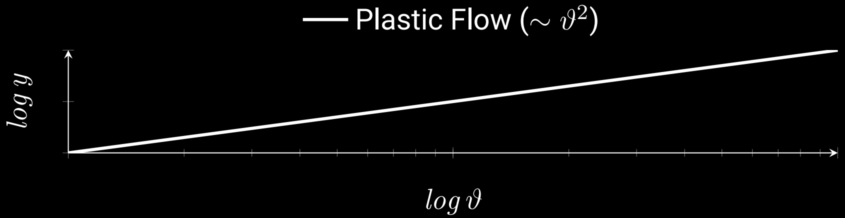
- Cantilever beam with a prescribed displacement at the tip $EA = 10$
- Simple yield function $\Phi = \left| \frac{N}{N_y \vartheta^2} \right| - 1$ and constant strain $\varepsilon = 0.11$



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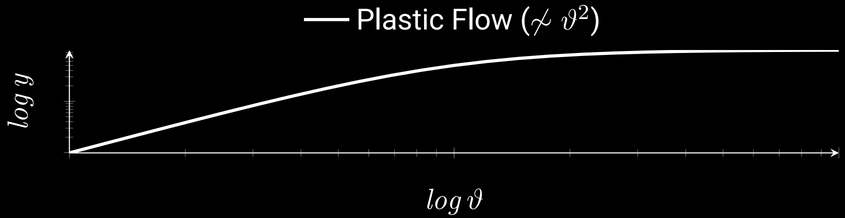

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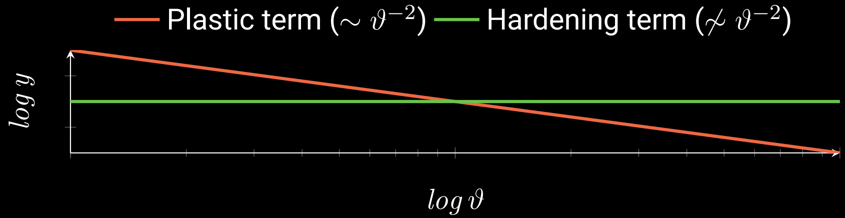

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- Hardening term is $\partial_{N_h}\boldsymbol{\Phi} \cdot H \cdot \partial_{N_h}\boldsymbol{\Phi}$
- The derivative $\partial_{N_h}\boldsymbol{\Phi}$ does not scale with ϑ
- Thus the hardening tensor should scale with ϑ^{-2}

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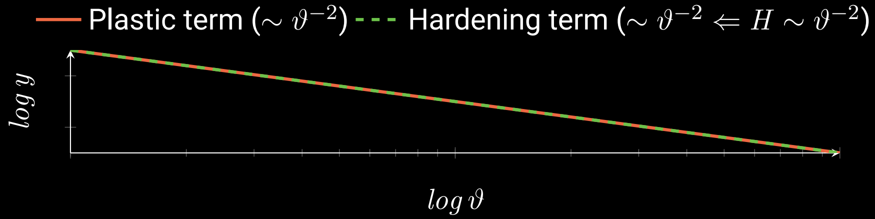

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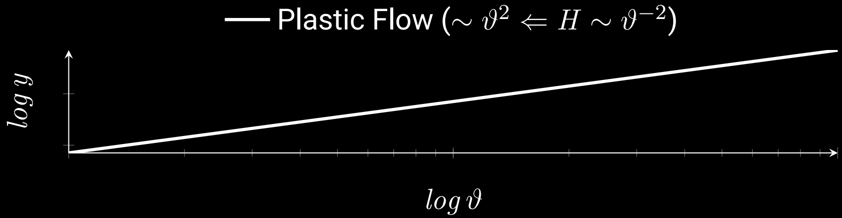

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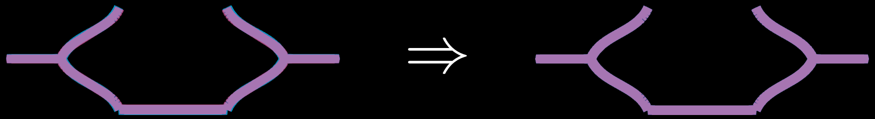

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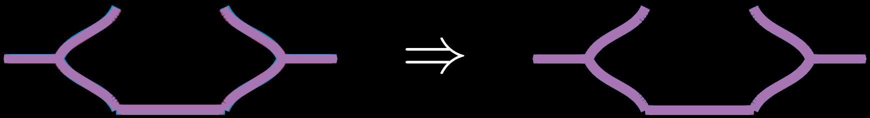
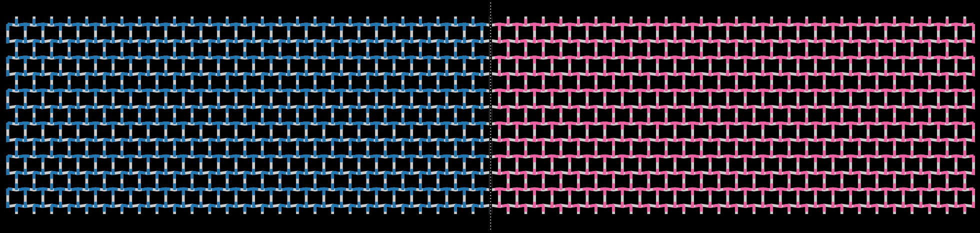
- Isotropic hardening of the form $\mathbf{\Phi} = |\ldots| - \Phi_y(1 + h_0)$ leads to scaling with the area inverse ϑ^{-2}



Conclusions and Outlook

- Kinematic hardening plasticity has been implemented for beam elements



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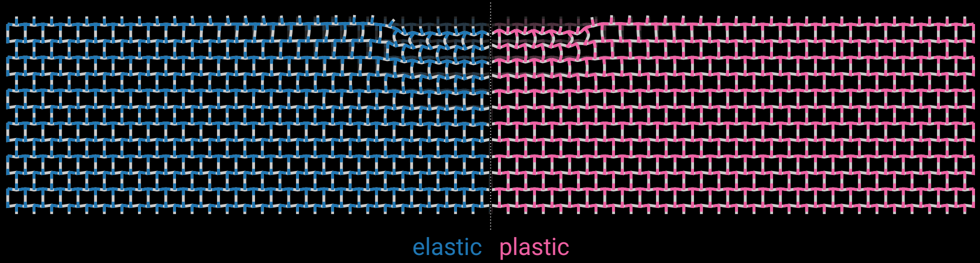

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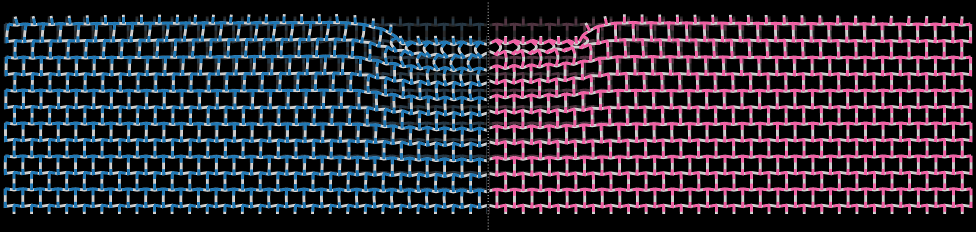

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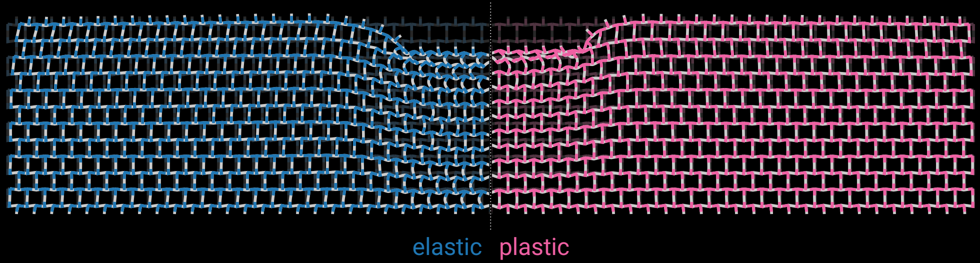

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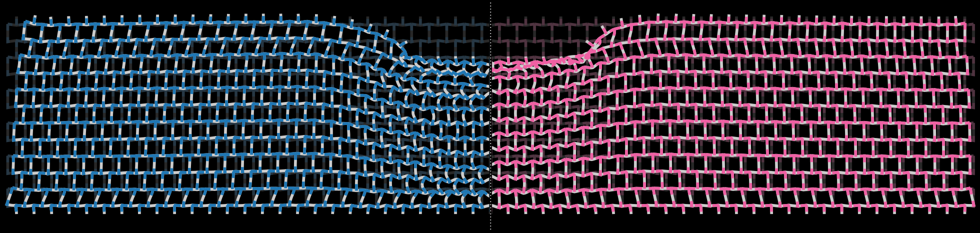
- The next step is the implementation of contact between the beams



Thank you!

Questions?

References I

[1] T. Gärtner, S. J. van den Boom, J. Weerheijm, and L. J. Sluys. "Geometric effects on impact mitigation in architected auxetic metamaterials". In: *Mechanics of Materials* 191 (2024).

[2] Ludwig Herrnböck, Ajeet Kumar, and Paul Steinmann. "Geometrically exact elastoplastic rods: determination of yield surface in terms of stress resultants". In: *Computational Mechanics* 67.3 (2021).

[3] Ludwig Herrnböck, Ajeet Kumar, and Paul Steinmann. "Two-scale off-and online approaches to geometrically exact elastoplastic rods". In: *Computational Mechanics* 71.1 (2022).

[4] Smriti, Ajeet Kumar, Alexander Großmann, and Paul Steinmann. "A thermoelastoplastic theory for special Cosserat rods". In: *Mathematics and Mechanics of Solids* 24.3 (2018).

References II

[5] Smriti, Ajeet Kumar, and Paul Steinmann. "A finite element formulation for a direct approach to elastoplasticity in special Cosserat rods". In: *International Journal for Numerical Methods in Engineering* 122.5 (2020).

[6] Michael A. Crisfield and Gordan Jelenić. "Objectivity of strain measures in the geometrically exact three-dimensional beam theory and its finite-element implementation". In: *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 455.1983 (1999).

[7] J. C. Simo and L. Vu-Quoc. "A three-dimensional finite-strain rod model. part II: Computational aspects". In: *Computer Methods in Applied Mechanics and Engineering* 58.1 (1986).